Summer Work for students entering Precalculus

Congratulations on your successful completion of Algebra I, Geometry, and Algebra II!

This summer packet is designed to help you review important Algebra II concepts in preparation for Precalculus. The topics covered in this packet are: Coordinate Geometry, Graphing Linear Equations/Inequalities, Writing Linear Equations, Solving Linear Systems, Radical Expressions, Complex Numbers, Solving Quadratic Equations, Parabolas and Logarithms. **You need to understand each of these concepts so that you will do well in Precal.**

In an organized fashion, show your work neatly on separate sheets of notebook paper. You may work with another rising Precal student, but the Honor Code applies: your work must be your own. I have included formulas, examples, and explanations at the end of this packet to help you. And, don’t forget [www.khanacademy.org](http://www.khanacademy.org) which is a wonderful website that explains mathematical topics through video. Just put in the topic and choose the one you need.

We strongly suggest that you do not begin this packet until mid-July. You will be tested on this material within the first few days of school, so it is important that these concepts are not forgotten over the course of the summer.

We look forward to a productive and successful year in Precalculus!
Section I: Distance, Midpoint, Pythagorean Theorem
1. Find the distance between the points (15, -8) and (17, 2).
2. Find the distance between the points (-11\sqrt{2}, 4) and (-3\sqrt{2}, -5).
3. Find the coordinates of the midpoint of the segment with endpoints (0, 6) and (-4, 13).
4. Find the coordinates of the midpoint of the segment with endpoints (1/4, -2) and (1/5, 4).
5. Given: A(-11, -9), M(6, -7). If M is the midpoint of segment AB, find the coordinates of B.
6. The endpoints of a diameter of a circle are (2, 7) and (6, 5). Find the radius of the circle.
7. The endpoints of a diameter of a circle are (2, 7) and (6, 5). Find the center of the circle.
8. The sides of a triangle are 20 cm, 99 cm, and 101 cm. Determine whether the triangle is right, acute, or obtuse.
9. The leg of a right triangle is 8; the hypotenuse is 12. Find the measure of the other leg.
10. In parallelogram ABCD, the coordinate of B is (5, -3) and the coordinate of D is (-2, -7). Find the midpoint of segment AC.
11. The length of a rectangle is 7 inches and the width is 5 inches. Find the length of the diagonal.
12. The length of the diagonal of a square is $\sqrt{64}$ ft. Find the length of the side of the square.

Section II: Graphing Linear Equations and Inequalities
Without the use of a graphing calculator, graph the following linear equations or inequalities:
1. $y = -\frac{3}{5}x + 2$
2. $y < -10 + 3x$
3. $y = -2$
4. $x > 4$
5. $4x - 3y \leq 9$
6. $2(x + 5) = 3 - 2y$
7. $3x + 18y > 2x + 17y$
8. $3x - 7 - 2y \geq x - y$

Section III: Slope & Writing Linear Equations
1. Find the slope between the points (-3, 5) and (1, 2).
2. Determine the value of a that makes the slope of the line through the two given points equal to the given value of $m$... (4, -3) and (2, a); $m = \frac{1}{4}$
3. Find the x- and y-intercepts of the line $4x - 9y = 18$.
4. Write an equation of a vertical line passing through the point (34, -65).
5. Find the equation of a line that has an x-intercept of -5 and a y-intercept of 13.
6. Write a linear equation in slope intercept form passing through the points (2, -2) and (-4, 13).
7. Write an equation of a line in standard form passing through the points (-2, 4) and (1, -5).
8. Write an equation of a line in standard form passing through the points (0, -7) and (7, -4).
9. Write a linear equation in slope intercept form passing through (3, 10) parallel to $3y - x = 4$.
10. Write an equation of a line in standard form passing through (1, -4) perpendicular to $x + 4y = -1$.
11. Write an equation of a line passing through the point (-6, 15) perpendicular to $x = -7$.
12. Determine the value of $k$ so that the line through the points (-2, 3) and (6, $k$) has y-intercept 4.

Section IV: Solving Linear Systems
Solve by Elimination (also known as the “Addition Method”):
1. $4x + y = 5$
   $x - 2y = 8$
2. $4m + 5n = 3$
   $5m + 6n = 2$

Solve by Substitution:
3. $y = 2x - 3$
   $-2x + y = 5$
4. $3b - 6a = 0$
   $4a + 3b = 26$
**Solve by Graphing:**
5. \(2y - x = 0\)
6. \(y = 1 - x\) **You should be familiar with graphing and solving systems on the calculator, also. Check your answers to this section with a calculator.**
\(\frac{1}{2}x - y = 0\)
\(2x + y = 9\)

**Word Problems: (Use any method)**
7. Eric bought 3 rolls of film and 1 battery for $11. The next day he purchased 2 rolls of film and 3 batteries for $12. Find the cost of one roll of film and one battery.

8. There are 8 more quarters than dimes in a parking meter. Three times the number of dimes is 1 less than twice the number of quarters. Find the number of dimes and quarters.

**Section V: Radical Expressions & Complex Numbers**
**Simplify:**
1. \(\sqrt{24a^3}\)
2. \(\sqrt{-72x^4y^3}\)
3. \((2 + 3i) - (5 - 7i)\)
4. \((4 - 3i)(5 + 2i)\)
5. \((2i\sqrt{5})^2\)
6. \(\frac{2i + 1}{3i}\)
7. \(\frac{2}{2 + \sqrt{5}}\)
8. \(\frac{4 + 5i}{1 + i}\)
9. \(\frac{\sqrt[3]{3a^7b^6c^5}}{\sqrt[3]{24a^2bc}}\)
10. \(2\sqrt{54} - 6\sqrt{\frac{2}{3}} - \sqrt{96}\)
11. \(3\sqrt{-50} + 5\sqrt{-18} - 6\sqrt{-200}\)
12. \(\frac{1}{5}(-10 + \sqrt{-125})\)
13. \((5 - \sqrt{-125}) - (4 - \sqrt{-20})\)
14. \((-3\sqrt{-5})(\sqrt{-20})\)
15. \((5 + 3i)^2\)
16. \(\frac{i + i^2 + i^3 + i^4}{1 + i}\)
Section VI: Solving Quadratic Equations

Solve by factoring:
1. \(5y = 10y^2\)
2. \(n^2 - 4n + 3 = 0\)
3. \(6y^2 + 2y - 4 = 0\)
4. \(2w^3 - 4w^2 - 16w = 0\)

Solve by using the quadratic formula:
5. \(2x^2 + 8x + 5 = 0\)
6. \(5t^2 = -12t - 8\)
7. \(3x(x - 4) = 5(2 - x)\)

Solve. Use the most appropriate method.
8. \(y^2 + 4\sqrt{2}y + 16 = 0\)
9. \(\frac{x-1}{2} - \frac{5}{2} = \frac{2}{1-x}\)
10. \(\left(\frac{x+1}{x+2}\right)^2 + 2\left(\frac{x+1}{x+2}\right) = 8\)
11. \(4x^2 = 11 + 4x\)
12. \(x^2 - 5x^2 - 6 = 0\)
13. \(x^2 + 6ix - 8 = 0\)
14. \(3x - 2\sqrt{3x} = 8\)

VII. Give the vertex, x intercepts, and y intercept for each parabola. Graph each.
1) \(y = 9x^2 - 4\)
2) \(y = x^2 - x - 12\)
3) \(y = \frac{1}{4}(x - 4)^2 + 1\)
4) \(y = -(x - 3)^2\)

VIII. Writing Equations of Parabolas

Find the quadratic function in the form \(y = a(x - h)^2 + k\) whose graph satisfies the given conditions.
1. has vertex (1,3) and passes through (4, 21)
2. has vertex (-1,4) and passes through (5, -14)
3. has \(x = 3\) as its axis of symmetry and passes through the points (2,5) and (-1,-25)
   (Hint: Solve a system of equations in \(a\) and \(k\))

VIII. Evaluate:
1) \(\log_{10} 1000\)
2) \(\log_2 2\sqrt{2}\)
3) \(\log_6 (6^5)\)
4) \(\log_{10} 101\) lies between what two consecutive integers?
5) If \(g(x) = 16^x\), then \(g^{-1}(x) = \)

IX. Solve for \(x\):
1) \(\log_2 x = 3\)
2) \(\log_6 x = 2.5\)
3) \(\log_{\frac{1}{2}} x = -\frac{1}{2}\)
4) \(2 = \log_2 x\)
5) \(\log x 7 = 1\)
6) \(\log x 2 = 0\)
X. Express as a log of a single number or expression:

1) \( \log_a 5 - \log_a 4 \) \\
2) \( 2 \log_a 7 \) \\
3) \( -\frac{1}{6} \log_a \frac{1}{6} \) \\
4) \( \log_b 4 + \log_b 5 - \log_b 2 \) \\
5) \( \log_b x - 3 \log_b y \) \\
6) \( \frac{1}{2} (\log_b x - \log_b y) \)

XI. Express each logarithm in terms of \( \log_3 M \) and \( \log_3 N \).

1) \( \log_3 N^7 \) \\
2) \( \log_3 \left( \frac{M}{N^3} \right) \) \\
3) \( \log_3 \sqrt{M} \) \\
4) \( \log_3 \frac{1}{N \sqrt{N}} \)

XII. Evaluate:

1) \( 2 \log_{10} 5 + \log_{10} 4 \) \\
2) \( \log_4 3 - \log_4 48 \)

XIII. Solve each equation:

1) \( \log_a x = \frac{3}{2} \log_a 9 + \log_a 2 \) \\
2) \( \log_b (x^2 + 7) = \frac{2}{3} \log_b 64 \) \\
3) \( \log_a (3x + 5) - \log_a (x - 5) = \log_a 8 \) \\
4) \( \log_3 (x + 2) + \log_3 6 = 3 \)

Formulas, Examples, and Explanations

This portion of the packet is designed to assist you with the problems. Here you will find formulas, and some examples & explanations. You will also want to look in your Algebra II notes from the previous year for help.

Section I:
To find the distance between two points \((x_1, y_1)\) and \((x_2, y_2)\) use the distance formula:

\[
d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}
\]

Example: Distance between \((5, 4)\) and \((-3, 0)\):

\[
d = \sqrt{(-3 - 5)^2 + (0 - 4)^2} = \sqrt{80} = 4\sqrt{5}
\]

To find the midpoint between two points \((x_1, y_1)\) and \((x_2, y_2)\) use the midpoint formula:

\[
\left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)
\]

The Pythagorean Theorem describes the relationship between the lengths of the sides in a right triangle, where \(a\) and \(b\) are the legs, and \(c\) is the hypotenuse:

\[
a^2 + b^2 = c^2
\]

Sections II & III:

<table>
<thead>
<tr>
<th>Formulas &amp; Concepts:</th>
<th>Forms of a linear equation:</th>
</tr>
</thead>
<tbody>
<tr>
<td>- Slope between two points: ( m = \frac{y_2 - y_1}{x_2 - x_1} )</td>
<td>- Point-Slope Form: ( y - y_1 = m(x - x_1) )</td>
</tr>
<tr>
<td>- Zero slope (0/0)… a horizontal line (( y = ? ))</td>
<td>- Slope-Intercept Form: ( y = mx + b ) (( m ) is slope; ( b ) is y-intercept)</td>
</tr>
<tr>
<td>- Undefined Slope (( \frac{a}{0} ))… a vertical line (( x = ? ))</td>
<td>- Standard Form: ( Ax + By = C ) (where ( A, B, ) &amp; ( C ) must be integers; ( A &gt; 0 )</td>
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</tbody>
</table>
- Parallel lines have equal slopes
- Perpendicular lines have opposite reciprocal slopes (example: \( m_1 = \frac{3}{2}, m_2 = -\frac{2}{3}; \) slopes are \( \perp \) )
- To find the x-intercept, plug in 0 for y and solve for x.
- To find the y-intercept, plug in 0 for x and solve for y.

**Examples:**

1. A) Graph the line \( 4x + 3y = -9 \).
   B) Graph the line \( x = 6 \).
   
   A) Solve for y to put the equation into slope-intercept form...
   \[ y = -\frac{4}{3}x - 3 \]
   Notice that \(-\frac{4}{3}\) is the slope and -3 is the y-intercept. Starting at the y-intercept of -3, you must “rise over run” \(-\frac{4}{3}\): move down 4 and right 3, then make your next point. Do this a few times in both directions, then sketch the line.

   B) The line \( x = 6 \) has an undefined slope, so we must sketch a vertical line passing through \( x = 6 \).

2. Graph in the inequality \( 5x - y > -1 \)

   Solve for \( y \) to put the line in slope-intercept form. Remember when you multiply or divide by a negative, you must flip the inequality...
   \[ y < 5x + 1 \]
   Remember that \( a \leq \) or \( a \geq \) indicates a dotted line, whereas \( a < \) or \( a > \) indicates a solid line. Graph a dotted line, starting from the y-intercept of +1. Since the slope is 5, you will rise 5 and run 1.
   Lastly, “less than” means to shade below the line.

**Examples (continued):**

3. Write a linear equation in standard form passing through the points (4, 1) and (0, -4).
   - Find the slope using the slope formula... \( m = \frac{3}{4} \)
   - Use point-slope form... choose either of the two given points.
   \[ y - 1 = \frac{3}{4}(x - 4) \]
   - Put the equation into standard form, eliminating all the fractions, and making A positive.
   \[ \frac{3}{4}x + y = 5 \]
   - You can check your answer by plugging in both of the given coordinates for \( x \) and \( y \).

4. Write a linear equation in standard form with an x-intercept of 8, perpendicular to the line \( x - 5y = 19 \)
   - If the x-intercept is 8, then we have a coordinate (8, 0).
   - Perpendicular lines have opposite reciprocal slopes, so we should find the slope of \( \frac{3}{5} \)...
   - Do this by putting the equation into slope-intercept form...
   - Since this line has a \(-\frac{3}{5}\) slope, our line will have a +5 slope.
   - Use the point (8,0) and the slope \( m = -\frac{3}{5} \) in point-slope form...
   \[ y - 0 = -\frac{3}{5}(x - 8) \]
   - Put the equation into standard form
   \[ \frac{5}{3}x + y = \frac{20}{3} \]
   \[ x + 5y = 8 \]
Section IV:
When we solve a system, we are finding the coordinate in which two lines intersect. We could also say that we are finding the ordered pair \((x, y)\) that makes both equations true. There are four methods for solving systems with which you should be familiar: Graphing, Elimination (or “addition”), Substitution, and using the TI-83 Calculator.

Examples:
1. Solve by Elimination:
\[
\begin{align*}
2x - y &= 6 \\
3x + 5y &= 22
\end{align*}
\]
First, eliminate a variable by multiplying either equation through by a constant... multiplying the top equation by +5 makes the most sense, in order to eliminate the y's...
\[
\begin{align*}
10x - 5y &= 30 \\
3x + 5y &= 22
\end{align*}
\]
Now add both equations to eliminate the y's, and solve for x...
\[
\begin{align*}
10x - 5y + 3x + 5y &= 30 + 22 \\
13x &= 52 \\
x &= 4
\end{align*}
\]
Now plug in 4 for x in either of the original equations to solve for y...
\[
2(4) - y = 6... \quad y = 2
\]
**Solution:** \((4, 2)\)

2. Solve by Substitution:
\[
\begin{align*}
2k - 3m &= 4 \\
m &= \frac{2}{3}k - \frac{4}{3}
\end{align*}
\]
Notice that the second equation is already solved for m; simply substitute \(\frac{2}{3}k - \frac{4}{3}\) for m into the first equation...
\[
2k - 3\left(\frac{2}{3}k - \frac{4}{3}\right) = 4
\]
Now solve for k...
\[
2k - 2k + 4 = 4 \\
4 = 4
\]
In this problem, the variable k cancelled, and since 4 = 4 is a true statement, any x and any y will make this system true. We write a solution that indicates all \((x,y)\)'s on the line are solutions.
\[\{x,y\} = 2k - 3m = 4\]
**Solution:** \((x,y)\) \(2k - 3m = 4\)

(Footnote: A false statement indicates no solution.)

3. Solve by Graphing Calculator:
\[
\begin{align*}
x - 8y &= 24 \\
y &= -\frac{3}{4}x + 12
\end{align*}
\]
To type an equation into the calculator, you must solve for y.
\[
x - 8y = 24... \quad y = \frac{1}{6}x - 3
\]
Type the equations into \(Y_1 \& Y_2\), and graph (Zoom Standard will reset your x- & y-axes.)
\[
\begin{align*}
10x - 5y &= 30 \\
3x + 5y &= 22
\end{align*}
\]
Notice that the lines intersect, but not on the screen. You must adjust your X-Max by choosing Window.
\[
\begin{align*}
x - 8y &= 24 \\
y &= -\frac{3}{4}x + 12
\end{align*}
\]
To find the intersection, go to 2nd Calc, then intersect. Press enter 3 times (1st curve, 2nd curve, guess) and your calculator will find the intersection.
\[
13x = 52 \\
x = 4
\]
**Solution:** \((17.143, -0.857)\)
Section V:
- A radical expression is not simplified if it contains factors that are perfect squares.
  \[ \sqrt{90a^2b^6} = \sqrt{9\cdot10\cdot\frac{a^2}{a^2}\cdot\frac{b^6}{b^6}} = 3ab\sqrt{10a} \]
- The imaginary number \( i \) allows us to take the square roots of negative numbers.
- Facts about imaginary numbers:
  \[ i^2 = -1 \] (and therefore) \[ \sqrt{-1} = i \]

Examples: 1) \[ \sqrt{-128x} + \sqrt{-18x} = \sqrt{-1\cdot64\cdot2\cdot x} + \sqrt{-1\cdot9\cdot2\cdot x} = 8i\sqrt{2x} + 3i\sqrt{2x} = 11i\sqrt{2x} \]
  2) \( (10 - 5i)(-2 + 3i) \) … Foil Method… \( = -20 + 30i + 10i - 15i^2 = -5 + 40i \) (+15)

- Recall that \( \sqrt{\cdot} \)’s or \( i \)’s cannot appear in the denominator.

Examples: 1) \[ \frac{3}{2i\sqrt{5}} \quad \frac{i\sqrt{5}}{i\sqrt{5}} = \frac{3i\sqrt{5}}{2\cdot\sqrt{25}} = \frac{3i\sqrt{5}}{-2(5)} = \frac{-3i\sqrt{5}}{10} \]
  2) \( \frac{2 - i\sqrt{2}}{6 + i\sqrt{2}} \quad \frac{6 - i\sqrt{2}}{6 - i\sqrt{2}} = \frac{12 - 2i\sqrt{2} - 6i\sqrt{2} + 2i^2}{36 - 6i\sqrt{2} + 6i\sqrt{2} - 2i^2} = \frac{12 - 8i\sqrt{2} - 2}{36 - 2i^2} = \frac{10 - 8i\sqrt{2}}{38} = \frac{5 - 4i\sqrt{2}}{19} \]

↑ “conjugate”

Section VI:
- Quadratic Equations can be solved many ways. The two reviewed in this packet are:
  1. Solving by factoring
  2. The Quadratic Formula

Examples: 1) Solve by factoring:
  \[ 6x^2 - 11x = 10 \]
  \[ 6x^2 - 11x - 10 = 0 \]
  \[ (3x + 2)(2x - 5) = 0 \]
  \[ 3x + 2 = 0 \] or \[ 2x - 5 = 0 \]
  \[ x = -\frac{2}{3} \] or \[ x = \frac{5}{2} \]

2) Solve by factoring:
  \[ 12x = 44x^2 \]
  \[ 12x - 44x^2 = 0 \]
  \[ 4x(3 - 11x) = 0 \]
  \[ 4x = 0 \] or \[ 3 - 11x = 0 \]
  \[ x = 0 \] or \[ x = \frac{3}{11} \]

3) Solve using the Quadratic Formula:
  \[ 3x^2 - 8x + 14 = 0 \]
  \[ x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \]
  \[ x = \frac{8 \pm \sqrt{(-8)^2 - 4\cdot3\cdot14}}{2(3)} \]
  \[ x = \frac{8 \pm 2\sqrt{26}}{6} = \frac{4 \pm \sqrt{26}}{3} \]
Section VII: Graphing Parabolas

Two Forms of the parabola:

- **Standard Form:** \( y = ax^2 + bx + c \)
- **Vertex Form:** \( y = a(x - h)^2 + k \)

To graph a parabola, you need (as shown above):

A. **The vertex** of the parabola - the turning point of the parabola; it may be either a maximum or a minimum depending on the value of “\( a \)“.

B. **The roots** of the parabola - the place or places where the parabola crosses the x-axis; there can be 0, 1, or 2 roots.

C. **The y-intercept** of the parabola - the place where the parabola crosses the y-axis; there is always one when the parabola represents a function which means it opens either up or down.

Sections VIII – XIII: Logarithms

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**The Relationship**

\[ y = b^x \quad \text{is equivalent to} \quad \log_b(y) = x \]

(means the exact same thing as)

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**Example 1.** Write in exponential form: \( \log_2{32} = 5 \).

**Answer.** \( 2^5 = 32 \).

**Example 2.** Write in logarithmic form: \( 4^{-2} = \frac{1}{16} \).

**Answer.** \( \log_4{\frac{1}{16}} = \frac{1}{9} \).
Example 3. Evaluate $\log_3 \frac{1}{9}$

Answer. $\frac{1}{9}$ is equal to 3 with what exponent? $\frac{1}{9} = 3^{-2}$.

$\log_3 \frac{1}{9} = \log_3 3^{-2} = -2$.

Example 4. $\log_2 .25 = ?$

Answer. $.25 = \frac{1}{4} = 2^{-2}$. Therefore,

$\log_2 .25 = \log_2 2^{-2} = -2$.

Example 5. $\log_3 \sqrt[3]{3} = ?$

Answer. $\sqrt[3]{3} = 3^{1/3}$.

$\log_3 \sqrt[3]{3} = \log_3 3^{1/3} = 1/3$.

The three laws of logarithms

1. $\log_b xy = \log_b x + \log_b y$
   "The logarithm of a product is equal to the sum of the logarithms of each factor."

2. $\log_b \frac{x}{y} = \log_b x - \log_b y$
   "The logarithm of a quotient is equal to the logarithm of the numerator minus the logarithm of the denominator."

3. $\log_b x^n = n \log_b x$
   "The logarithm of $x$ with a rational exponent is equal to the exponent times the logarithm."

Example Apply the laws of logarithms to $\log \frac{abc^2}{d^3}$.

Answer.

$$\log \frac{abc^2}{d^3} = \log (abc^2) - \log d^3$$
$$= \log a + \log b + \log c^2 - \log d^3$$
$$= \log a + \log b + 2 \log c - 3 \log d,$$