Summer Work – Preparation for Algebra III/Trig

This assignment is to be completed after August 1 for best results.

This work is due the first Monday after returning to school. You will earn your first grade on this work.

You will be able to ask questions in class, but is should be complete.

1. **COMPLETE WORK NEATLY ON LINED PAPER!**
2. **PUT YOUR FINAL ANSWER ON ANSWER SHEET AT THE END OF THIS PACKET.**
3. Graphs should be completed on graph paper.
4. **Know this material.** We will expand on it.
5. You may work together and get help on this assignment. Just don’t copy someone’s work.
6. Odd answers are included. YOU MAY USE A CALCULATOR.

This summer packet is designed to help you review some of the important concepts of Algebra II, in preparation for Algebra III. The topics covered in this packet are: Graphing Linear Equations/Inequalities, Writing Linear Equations, Solving Linear Systems, Monomials/Polynomials, Radical Expressions, Complex Numbers, and Solving Quadratic Equations.
Section I: Graphing Linear Equations and Inequalities

On graph paper, graph the following linear equations or inequalities without a graphing calculator:

1. \( y = -\frac{3}{5}x + 2 \)  
2. \( y < -10 + 3x \)  
3. \( y = -2 \)  
4. \( x > 4 \)

5. \( 4x - 3y < 9 \)  
6. \( 2(x + 5) = 3 - 2y \)  
7. \( 3x + 18y > 2x + 17y \)  
8. \( 3x - 7 - 2y > x - y \)

Section II: Slope & Writing Linear Equations

1. Find the slope between the points (-3, 5) and (1, 2).
2. Find the slope between the points (13, 52) and (13, -27).
3. Find the slope between the points (-6, -19) and (1, -19).
4. Write linear equation in SLOPE INTERCEPT form with \( m = -2 \) and \( b = -3 \).
5. Write linear equation in STANDARD form with \( m = -2 \) and \( b = -3 \).
6. Find the slope and y-intercept of the line \( 3(y - 2x) = 4x + 2 \).
7. Find the x- and y-intercepts of the line \( 4x - 9y = 18 \).
8. Write an equation of a vertical line passing through the point (34, -65).
9. Write an equation of a horizontal line passing through the point (12, -2).
10. A line has an x-intercept of -5 and a y-intercept of 13. Find the equation of this line in slope intercept form.
11. Write a linear equation in SLOPE INTERCEPT form passing through the points (2, -2) and (4, 13).
12. Write an equation of a line in STANDARD form passing through the points (-2, 4) and (1, -5).
13. Write a linear equation SLOPE INTERCEPT form passing through (3, 10) parallel to \( 3y - x = 4 \).
14. Write an equation of a line in STANDARD form passing through (1, -4) perpendicular to \( x + 4y = -1 \).
15. Write an equation of a line passing through the point (-6, 15) perpendicular to \( x = -7 \).

Section III: Solving Linear Systems

Solve by Elimination (also known as the “Addition Method”):

1. \( 4x + y = 5 \)  
2. \( 4m + 5n = 3 \)  
3. \( x - 2y = 8 \)  
4. \( 5m + 6n = 2 \)

Solve by Substitution:

3. \( y = 2x - 3 \)  
4. \( 3b - 6a = 0 \)  
-2x + y = 5  
4a + 3b = 26

Solve by Graphing:

5. \( 2y - x = 0 \)  
6. \( y = 1 - x \)  
\( \frac{1}{2}x - y = 0 \)  
2x + y = 9
Section IV: Monomials & Polynomials; Rules for Exponents

Simplify:

1. \((3x^2)^2(-2x^3) - 2x^2 + 5x^7\)
2. \(\left(\frac{3}{2}y\right)^3\left(\frac{4}{3}y^3\right)^2\)
3. \((3x^{-2}y^3)(5xy^{-8})^{-1}\)
4. \(\frac{(2ab^{-1})(4a^2b^3)^{-2}}{(a^{-2}b^3)^{-3}}\)
5. \((-6n - 13n^2) + (-3n + 9n^2)\)
6. \((5m^2 - 2mp - 6p^2) - (-3m^2 + 5mp + p^2)\)
7. \(-6a^2w(a^3w - aw^4)\)
8. \((v^2 - 6)(v^2 + 4)\)
9. \((x^2 + 5y)^2\)
10. \(u(u - 6)(u - 3)\)

Section V: Radical Expressions & Complex Numbers

Simplify:

1. \(\sqrt{24a^5}\)
2. \(\sqrt{-72x^4y^5}\)
3. \(\sqrt[3]{24a^3b^{13}}\)
4. \((2 + 3i) - (5 - 7i)\)
5. \((4 - 3i)(5 + 2i)\)
6. \((2i\sqrt{5})^2\)
7. \(\frac{2i + 1}{3i}\)
8. \(\frac{2}{2 + \sqrt{5}}\)
9. \(\frac{4 + 5i}{1 + i}\)

Section VI: Solving Quadratic Equations

Solve by factoring:

1. \(5y = 10y^2\)
2. \(n^2 - 4n + 3 = 0\)
3. \(3x^2 + 8x = 3\)
4. \(y(y - 3) = 12 + y\)
5. \(2w^3 - 4w^2 - 16w = 0\)
6. \(x^2 + 6x + 4 = 0\)
7. \(2x(x + 4) = -5\)
8. \(5t^2 = -12t - 8\)
9. \(h^2 + 7 = 4h\)
10. \(3x(x - 4) = 5(2 - x)\)
Formulas, Examples, and Explanations

This portion of the packet is designed to assist you with the problems. Here you will find formulas, and some examples & explanations. You will also want to look in your Algebra II notes from the previous year for help.

Sections I & II:

Formulas & Concepts:
- Slope between two points: \( m = \frac{y_2 - y_1}{x_2 - x_1} \)
- Zero slope (\( \frac{0}{b} \))… a horizontal line (Example: \( y = 3 \) is a horizontal line passing through 3)
- Undefined Slope (\( \frac{a}{0} \))… a vertical line (Example: \( x = -2 \) is a vertical line passing through -2)
- Parallel lines have equal slopes (Example: \( y = \frac{1}{3}x - 4 \) and \( y = \frac{1}{3}x + 9 \))
- Perpendicular lines have opposite reciprocal slopes (Example: \( y = -3x - 2 \) and \( y = \frac{1}{3}x + 1 \))
- To find the x-intercept, plug in 0 for y and solve for x.
- To find the y-intercept, plug in 0 for x and solve for y.

Forms of a linear equation:
- Point-Slope Form: \( y - y_1 = m(x - x_1) \)
- Slope-Intercept Form: \( y = mx + b \) (\( m \) is slope; \( b \) is y-intercept)
- Standard Form: \( Ax + By = C \) (where \( A, B, \) & \( C \) must be integers, and \( A > 0 \))

Examples:
1. A) Graph the line \( 4x + 3y = -9 \).
   B) Graph the line \( x = 6 \).

A) Solve for y to put the equation into slope-intercept form...
   \[ y = -\frac{4}{3}x - 3 \]
   Notice that \( -\frac{4}{3} \) is the slope and -3 is the y-intercept. Starting at the y-intercept of -3, you must “rise over run” \( -\frac{4}{3} \): move down 4 and right 3, then make your next point. Do this a few times in both directions, then sketch the line.

B) The line \( x = 6 \) has undefined slope, so we must sketch a vertical line passing through \( x = 6 \).
2. Graph in the inequality $5x - y > -1$

Solve for $y$ to put the line in slope-intercept form. Remember when you multiply or divide by a negative, you must flip the inequality...

$$y < 5x + 1$$

Recall that $<$ or $>$ indicates a dotted line, whereas $\leq$ or $\geq$ indicates a solid line. Graph a dotted line, starting from the y-intercept of +1. Since the slope is 5, you will rise 5 and run 1. Lastly, “less than” means to shade below the line...

3. Write a linear equation in standard form passing through the points (4, 1) and (0, -4).

- Find the slope using the slope formula... $m = \frac{5}{4}$.
- Use point-slope form... choose either of the two given points.
  $$y - 1 = \frac{5}{4} (x - 4)$$
- Put the equation into standard form, eliminating all the fractions, and making $A$ positive.
  $$y = \frac{5}{4} x - 5$$
  $$y = \frac{5}{4} x - 4 \quad \text{...slope-intercept form}$$
  $$\frac{5}{4}x + y = -4$$
  $$5x - 4y = 16 \quad \text{... standard form}$$
- You can check your answer by plugging in both of the given coordinates for $x$ and $y$.

4. Write a linear equation in standard form with an x-intercept of 8, perpendicular to the line $x - 5y = 19$

- If the x-intercept is 8, then we have a coordinate (8, 0).
- Perpendicular lines have opposite reciprocal slopes, so we should find the slope of $x - 5y = 19$.
- Do this by putting the equation into slope intercept form... $y = -\frac{1}{5} x - \frac{19}{5}$ OR if the line is in standard form ($Ax + By = C$) the slope of the line is $-A/B$.
- Since this line has a $-\frac{1}{5}$ slope, our line will have a +5 slope.
- Use the point (8,0) and the slope $m = -\frac{1}{5}$ in point-slope form...
  $$y - 0 = -\frac{1}{5} (x - 8)$$
- Put the equation into standard form
  $$y = -\frac{1}{5} x + \frac{8}{5}$$
  $$\frac{1}{5}x + y = \frac{8}{5}$$
  $$x + 5y = 8$$
Formulas, Examples, and Explanations (continued)

Section III:
When we solve a system, we are finding the coordinate in which two lines intersect. We could also say that we are finding the ordered pair \((x, y)\) that makes both equations true. There are four methods for solving systems with which you should be familiar: Graphing, Elimination (or “addition”), Substitution, and using the TI-83 Calculator.

Examples:
1. Solve by Elimination:
\[
\begin{align*}
2x - y &= 6 \\
3x + 5y &= 22
\end{align*}
\]
First, eliminate a variable by multiplying either equation through by a constant...
Multiplying the top equation by +5 makes the most sense, in order to eliminate the y’s...
\[
\begin{align*}
10x - 5y &= 30 \\
3x + 5y &= 22
\end{align*}
\]
Now add both equations to eliminate the y’s, and solve for \(x\)...
\[
\begin{align*}
10x - 5y &= 30 \\
+ 3x + 5y &= 22 \\
13x &= 52 \\
x &= 4
\end{align*}
\]
Now plug in 4 for \(x\) in either of the original equations to solve for \(y\)...\[
\begin{align*}
2(4) - y &= 6 \\
y &= 2
\end{align*}
\]
Solution: \((4, 2)\)

2. Solve by Substitution:
\[
\begin{align*}
2k - 3m &= 4 \\
m &= \frac{2}{3}k - \frac{4}{3}
\end{align*}
\]
Notice that the second equation is already solved for \(m\); simply substitute \(\frac{2}{3}k - \frac{4}{3}\) for \(m\) into the first equation...
\[
2k - 3\left(\frac{2}{3}k - \frac{4}{3}\right) = 4
\]
Now solve for \(k\)...
\[
\begin{align*}
2k - 2k + 4 &= 4 \\
4 &= 4
\end{align*}
\]
In this problem, the variable \(k\) cancelled, and since 4 = 4 is a true statement, any \(x\) and any \(y\) will make this system true.
We write a solution that indicates all \((x,y)’s\) on the line are solutions.
Solution: \({(x,y)| 2k - 3m = 4}\)

3. Solve by Graphing Calculator:
\[
\begin{align*}
x - 8y &= 24 \\
y &= -\frac{3}{4}x + 12
\end{align*}
\]
To type an equation into the calculator, you must solve for \(y\).\[
\begin{align*}
x - 8y &= 24... \\
y &= \frac{1}{8}x - 3
\end{align*}
\]
Type the equations into Y1 & Y2, and graph (Zoom Standard will reset your x- & y-axes.) Notice that the lines intersect, but not on the screen. You must adjust your X-Max by choosing Window.
To find the intersection, go to 2nd Calc, then intersect. Press enter 3 times (1st curve, 2nd curve, guess) and your calculator will find the intersection.
Solution: \((17.143, -0.857)\)

Section IV:
- Adding/Subtracting Monomials: \(2x^7 - 6x^7 + 12x^7 = 8x^7\) … The three terms are “like terms”; the coefficients are added/subtracted, but the exponents remain the same
- Multiplying Monomials: \((3x^2y)(-2x^3y^2)(10x^4y^5) = -60x^9y^8\) … The coefficients are multiplied; the exponents are added
- Raising a Monomial to an Exponent: \((3a^5bc^{10})^3 = 27a^{15}b^3c^{30}\) … The coefficient is raised to the power; the exponents are multiplied
- Dividing Monomials: \(\frac{2x^3y^7z}{14x^5y^4z^6} = \frac{y^3}{7x^2z^5}\) … The coefficients are reduced; the exponents are subtracted.
Formulas, Examples, and Explanations (continued)

- **Negative Exponents:**
  \[
  \frac{(2a)^3 b^4}{3(ab)^5} = \frac{2^3 a^3 b^4}{3 a^{-5} b^{-5}} = \frac{a^3 b^4}{3 \cdot 2^3 \cdot a^3} = \frac{a^9 b^9}{24} \]
  ... Step 1: Parentheses are eliminated;
  Step 2: Negative exponents are made positive by switching their placement from the numerator to the denominator, or vice versa; Step 3: Expression is simplified by adding or subtracting exponents

- **Polynomial Addition:**
  \[(6x - 3x^3) + (2x + 4x^2 - 5x^3) = -8x^3 + 4x^2 + 8x \quad \text{... Combine like terms.}

- **Polynomial Subtraction:**
  \[(6x - 3x^3) - (2x + 4x^2 - 5x^3) = 6x - 3x^3 - 2x - 4x^2 + 5x^3 = 2x^3 - 4x^2 + 4x \quad \text{...Distribute the negative, then combine like terms.}

- **Polynomial Multiplication:**
  \[(2x + 3)(4x^2 + x) = 8x^3 + 2x^2 + 12x^2 + 3x = 8x^3 + 14x^2 + 3x \quad \text{...Use the FOIL method for binomials}

Section V:

- A radical expression is not simplified if it contains factors that are perfect squares.

  **Example:**
  \[
  \sqrt{90a^5b^6} = \sqrt[6]{9 \cdot 10 \cdot a^3 \cdot a \cdot b^5} = 3ab\sqrt[6]{10}
  \]
  - The imaginary number \(i\) allows us to take the square roots of negative numbers.
  - Facts about imaginary numbers:
    \[
    i^2 = -1 \quad \text{and therefore} \quad \sqrt{-1} = i
    \]

  **Examples:**
  1) \(\sqrt{-128x} + \sqrt{-18x} = \sqrt{-1 \cdot 64 \cdot 2 \cdot x} + \sqrt{-1 \cdot 9 \cdot 2 \cdot x} = 8i\sqrt{2x} + 3i\sqrt{2x} = 11i\sqrt{2x}
  
  2) \((10 - 5i)(-2 + 3i)\) ... Foil Method ... \(-20 + 30i + 10i - 15i^2 = -5 + 40i \quad (+15)
  
- Recall that \(\sqrt{}\)'s or \(i\)'s cannot appear in the denominator.

  **Examples:**
  1) \(\frac{3}{2i\sqrt{5}} \cdot \frac{i\sqrt{5}}{i\sqrt{5}} = \frac{3i\sqrt{5}}{2i^2 \sqrt{5}} = \frac{3i\sqrt{5}}{-2(5)} = -\frac{3i\sqrt{5}}{10}
  
  2) \(\frac{6 - i\sqrt{2}}{6 + i\sqrt{2}} = \frac{36 - 6i\sqrt{2} + 6i\sqrt{2} - 2i^2}{36 - 6i\sqrt{2} + 6i\sqrt{2} - 2i^2} = \frac{12 - 8i\sqrt{2} - 2}{36 - 2i^2} = \frac{10 - 8i\sqrt{2}}{38} = \frac{5 - 4i\sqrt{2}}{19}
  
  "conjugate"

Section VI:

Quadratic Equations can be solved many ways. The two reviewed in this packet are:

1. Solving by factoring
2. The Quadratic Formula

**Examples:**

1) Solve by factoring:
   \[6x^2 - 11x = 10 \]
   \[6x^2 - 11x - 10 = 0 \]
   \[(3x + 2)(2x - 5) = 0 \]
   \[3x + 2 = 0 \quad \text{or} \quad 2x - 5 = 0 \]
   \[x = -\frac{2}{3} \quad \text{or} \quad x = \frac{5}{2} \]

2) Solve by factoring:
   \[12x = 44x^2 \]
   \[12x - 44x^2 = 0 \]
   \[4x(3 - 11x) = 0 \]
   \[4x = 0 \quad \text{or} \quad 3 - 11x = 0 \]
   \[x = 0 \quad \text{or} \quad x = \frac{3}{11} \]

3) Solve using the Quadratic Formula:
   \[3x^2 - 8x + 14 = 0 \]
   \[x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(8) \pm \sqrt{(-8)^2 - 4 \cdot 3 \cdot 14}}{2(3)} = \frac{8 \pm \sqrt{-104}}{6} = \frac{8 \pm 2i\sqrt{26}}{6} = \frac{4 \pm i\sqrt{26}}{3} \]
Answer Sheet (Evens will be graded):

Section I: Use graph paper.

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Section IV:

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Odd Numbered Solutions:

Section II
1. \(-\frac{3}{4}\)  
3. 0  
5. \(2x + y = -3\)  
7. \((\frac{9}{2}, 0); (0, -2)\)  
9. \(y = -2\)  
11. \(y = -\frac{5}{2}x + 3\)  
13. \(y = \frac{1}{3}x + 9\)  
15. \(y = 15\)

Section III
1. \((2, -3)\)  
3. \(\emptyset\)  
5. \(\{(x, y) | x - 2y = 0\}\)

Section IV
1. \(-13x^7 - 2x^2\)  
3. \(\frac{3y^{11}}{5x^3}\)  
5. \(-4n^2 - 9n\)  
7. \(-6a^5w^2 + 6a^3w^5\)  
9. \(x^4 + 10x^2y + 25y^2\)

Section V
1. \(2a\sqrt{6a}\)  
3. \(2ab^4\sqrt[3]{3a^2b}\)  
5. \(26 - 7i\)  
7. \(\frac{2 - i}{3}\)  
9. \(\frac{9 + i}{2}\)

Section VI
1. \(\{0, \frac{1}{2}\}\)  
3. \(\{-3, \frac{1}{3}\}\)  
5. \(\{-2, 0, 4\}\)  
7. \(\left\{-\frac{4 \pm \sqrt{6}}{2}\right\}\)  
9. \(\left\{2 \pm i\sqrt{3}\right\}\)